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OPTIMIZATION OF THE STRUCTURE OF ROLLED SHELLS

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1. In [1], the direct problem of determining the stressed state of a cylindrical tube prepared by rolling a thin flexible shell is considered. The elastiplastic model for deformation of these structures is the following closed set of equations:

$$\frac{\partial\sigma_{11}^0}{\partial\lambda_1}+\frac{\partial\sigma_{12}^0}{a_2\partial\lambda_2}+\frac{\sigma_{11}^0-\sigma_{22}^0}{a_2}=0, \quad (1.1)$$

$$\frac{\partial\sigma_{12}^0}{\partial\lambda_1}+\frac{\partial\sigma_{22}^0}{a_2\partial\lambda_2}+\frac{2\sigma_{12}^0}{a_2}=0;$$

$$\frac{\partial w_1^0}{\partial\lambda_1}=\frac{1-\nu}{2\mu}\sigma_{11}^0-\frac{\nu}{2\mu}\sigma_{22}^0, \quad (1.2)$$

$$\frac{\partial w_2^0}{a_2\partial\lambda_2}+\frac{w_1^0}{a_2}=\frac{1-\nu}{2\mu}\sigma_{22}^0-\frac{\nu}{2\mu}\sigma_{11}^0;$$

$$\frac{\partial w_2^0}{\partial\lambda_1}+\frac{\partial w_1^0}{a_2\partial\lambda_2}-\frac{w_2^0}{a_2}=\frac{\sigma_{12}^0}{\mu}+\Gamma(\sigma_{12}^0,\lambda_1,\lambda_2). \quad (1.3)$$

Here (λ_1, λ_2) is the orthogonal curvilinear coordinate system; line $\lambda_1 = \text{const}$ is directed along the contact of shell layers; $a_2 = \lambda_1 + \xi\lambda_2 + R_0 \cos\delta$ is Lamé parameter; $\xi = R_0 \sin\delta$; R_0 is tube internal radius; δ is slope of spiral λ_2 to circle $r = R_0$; w_i^0, σ_{ij}^0 $(i, j = 1, 2)$ are displacement vector components and the stress tensor in coordinates (λ_1, λ_2); μ is shear modulus; ν is Poisson's ratio. Set (1.1) are normal equilibrium equations in curvilinear coordinates; (1.2) are equations determining the elastic change in dimensions of an elementary volume in directions λ_1 and λ_2; (1.3) characterizes the overall shear strain of an element of the material; the first term in the right-hand part is elastic deformation of shell layers; Γ is slippage of layers over each other. This stressed-strained state depends markedly on the form of function Γ, i.e., on the conditions at the contacts between shell layers. This situation may be used for optimizing the structure as a whole.

Let the shell be intended for operation at high internal pressures when as a best performance criterion we take

$$p \to \max \quad (1.4)$$

(p is the value of internal pressure). It is noted that this criterion should be fulfilled with prescribed internal pressure, material parameters μ, ν, shell layer thickness $h = 2\pi\xi$, and fulfillment of certain inequalities guaranteeing material integrity. Thus, if Eq. (1.3) is excluded from the closed set (i.e., Γ is considered as a controlling function), then best performance condition (1.4) may be used in order to obtain equations closing set (1.1), (1.2). After solving it from Eq. (1.3), where displacements and stresses are already known, we determine function Γ, which provides fulfillment of criterion (1.4). This is the general scheme for solving the problem.

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2. We consider a situation when plastic deformation of the shell is impermissible:

$$\sqrt{(\sigma_{11}^0-\sigma_{22}^0)^2+4\sigma_{12}^{0^2}}\leqslant 2k=2\alpha\tau_s, \tag{2.1}$$

where τ_s is material elastic limit; $0 < \alpha < 1$ is safety factor; $k = \alpha\tau_s$. Best performance criterion (1.4) leads to the situation that in the whole region in (2.1) equality should be achieved. Then problem (1.1), (1.2), (2.1) (with a sign of equality) becomes statically determinable. With normal boundary conditions

$$\sigma_r|_{r=R_0}=-p,\quad \sigma_{r\theta}|_{r=R_0}=0; \tag{2.2}$$

$$\sigma_{11}^0=-p+k\ln\left(\frac{a_2^2+\xi^2}{R_0^2}\right)+\frac{2k\xi^2}{a_2^2+\xi^2}, \tag{2.3}$$

$$\sigma_{22}^0=-p+k\ln\left(\frac{a_2^2+\xi^2}{R_0^2}\right)+\frac{2ka_2^2}{a_2^2+\xi^2},\quad \sigma_{12}^0=-\frac{2k\xi a_2}{a_2^2+\xi^2},$$

or in polar coordinates (r, θ)

$$\sigma_r=-p+2k\ln\left(\frac{r}{R_0}\right),\quad \sigma_\theta=-p+2k\left(1+\ln\left(\frac{r}{R_0}\right)\right),\quad \sigma_{r\theta}=0. \tag{2.4}$$

Distribution (2.4) is well known as a solution of the problem of plastic deformation of a one-piece thin-walled tube [2]. A remarkable feature of the structure in question is the fact that precisely this stressed state is realized with elastic deformation (in (2.1) $\alpha < 1$).

Of course displacements will differ from both elastic and plastic displacements [2]. They are governed by elastic deformations of the structure as a whole and by slippages between layers. By substituting (2.3) in set (1.2) we obtain

$$\begin{gathered} w_1^0=C_3a_2+\frac{1-2\nu}{2\mu}ka_2\ln(a_2^2+\xi^2)+2\frac{1-\nu}{\mu}k\xi\,\mathrm{arctg}\left(\frac{a_2}{\xi}\right)+f_2(\lambda_2), \\ w_2^0=\frac{1-\nu}{\mu}\frac{k}{\xi}a_2^2-2\frac{1-\nu}{\mu}ka_2\,\mathrm{arctg}\left(\frac{a_2}{\xi}\right)+\frac{1-2\nu}{2\mu}k\xi\ln(a_2^2+\xi^2)-\int f_2(\lambda_2)\,d\lambda_2+f_1(\lambda_1)\end{gathered} \tag{2.5}$$

$\left(C_3=-\frac{1-2\nu}{2\mu}(p+2k\ln R_0)-\frac{1-\nu}{\mu}k,\ f_i(\lambda_i)\ (i=1,2)\right.$ are arbitrary functions). If there are no displacements of the outer tube boundary, then:

$$w_1^0|_{\lambda_1=\chi_1}=0,\quad w_2^0|_{\lambda_2=\chi_2}=0, \tag{2.6}$$

and functions $f_i(\lambda_i)$ are reduced to constants

$$\begin{gathered} C_2=f_2(\lambda_2)=-C_3A-\frac{1-2\nu}{2\mu}kA\ln(A^2+\xi^2)-2\frac{1-\nu}{\mu}k\xi\,\mathrm{arctg}\left(\frac{A}{\xi}\right), \\ C_1=f_1(\lambda_1)=\frac{C_2A}{\xi}-\frac{1-\nu}{\mu}\frac{k}{\xi}A^2+2\frac{1-\nu}{\mu}kA\,\mathrm{arctg}\left(\frac{A}{\xi}\right)-\frac{1-2\nu}{2\mu}k\xi\ln(A^2+\xi^2).\end{gathered} \tag{2.7}$$

Here $A = \sqrt{R^2 - \xi^2}$; R is tube external radius; χ_1 and χ_2 are notations for the outer boundary of the region.

Thus, expressions (2.3), (2.5), (2.7) give the solution for the original optimization problem, and according to (1.3) function Γ has the form

$$\Gamma=C_4a_2+\frac{C_5}{a_2}\quad \left(C_4=\frac{1-\nu}{\mu}\frac{k}{\xi},\quad C_5=C_3\xi-C_1+2C_4\xi^2\right). \tag{2.8}$$

Thus, in the optimum structure, the dependence of σ_{12}^0 on a_2 is governed by the last equality of (2.3), and Γ is governed by equality (2.8). If it is assumed that there is only a plastic lubricant, i.e., Γ may only depend on σ_{12}^0, then the last equation in (2.3) and (2.8) may be considered as a parametric relationship where a_2 enters into the role of a parameter. Then from (2.3) and (2.8) it follows that with corresponding ratios of shell layer thickness h and internal radius R_0 ($h/R_0 < 0.5$-1) behavior at the contact is close to "ideally plastic":

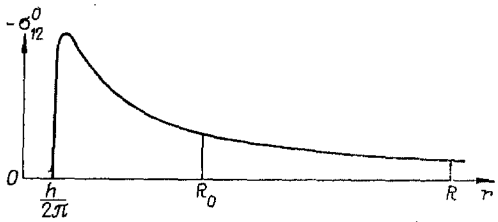

Fig. 1

σ_{12}^0 = const (Fig. 1). Therefore, if the condition at the contact is selected so that σ_{12}^0 = const, then the structure obtained with these ratios of parameters will not differ significantly from the optimum.

We consider this example. Let internal pressure p_0 ($p_0 = 0.99\tau_s$), and external pressure $q_0 = 0$ be prescribed, and only material elastic deformation be permissible. Then the minimum extrenal radius of a one-piece tube $R_1 = R_0/\sqrt{1 - p_0/\tau_s}$ ($R_1 = 10R_0$) . For a rolled shell an external radius $R_2 = R_0 \exp(p_0/(2\tau_s))$ ($R_2 \approx 1.64R_0$) appears adequate. It can be seen (Fig. 2) that in the second case a structure of considerably less thickness is required. As expected, this occurs uniformly throughout the thickness in contrast to a one-piece structure, where the material most loaded is only close to the inner edge (Fig. 3, σ_r', σ_θ', and σ_r'', σ_θ'' are stresses in a one-piece tube and in a rolled structure). It also follows from the last equations that with an unlimited increase in shell thickness its supporting capacity increases in an unlimited way, whereas in a one-piece tube an increase in thickness has practically no effect on supporting capacity.

3. In the structures studied there is external friction of layers. A situation is typical when external friction is dry. In equations of the continuum model, external dry friction enters as internal friction. Therefore taking account of the experience of solving the previous problem, we consider a situation when Eq. (1.3) is altered to

$$(\sigma_2 - \sigma_1)/2 = -f(\sigma_2 + \sigma_1)/2. \qquad (3.1)$$

Here $\sigma_2 > \sigma_1$ are principal values of stress tensor; f = const is the internal friction coefficient. Then Eqs. (1.1) and (3.1) form a closed set for stresses. Taking account of boundary conditions (2.2) its solution has the form

$$\sigma_{11}^0 = -p\left(\frac{a_2^2+\xi^2}{R_0^2}\right)^{-f/(1+f)}\left(1 - \frac{2f}{1+f}\,\frac{\xi^2}{a_2^2+\xi^2}\right), \qquad (3.2)$$

$$\sigma_{22}^0 = -p\left(\frac{a_2^2+\xi^2}{R_0^2}\right)^{-f/(1+f)}\left(1 - \frac{2f}{1+f}\,\frac{a_2^2}{a_2^2+\xi^2}\right),$$

$$\sigma_{12}^0 = -p\left(\frac{a_2^2+\xi^2}{R_0^2}\right)^{-f/(1+f)}\frac{2f}{1+f}\,\frac{\xi a_2}{a_2^2+\xi^2}$$

or in polar coordinates,

$$\sigma_r = -p\,(r/R_0)^{-2f/(1+f)}, \quad \sigma_\theta = -p\frac{1-f}{1+f}\left(\frac{r}{R_0}\right)^{-2f/(1+f)}, \quad \sigma_{r\theta} = 0. \qquad (3.3)$$

Stresses (3.2) satisfy the condition $\sigma_{12}^0 = \tan\varphi \cdot \sigma_{11}^0$, where

$$\tan\varphi = f\sin 2\varkappa/(1 + f\cos 2\varkappa) \text{ or } f = \sin\varphi/[\sin(2\varkappa - \varphi)] \qquad (3.4)$$

($\varkappa$ is angle between the circle r = const and contact lines of shell layers: $\tan\varkappa = \xi/a_2 = \xi/\sqrt{r^2 - \xi^2}$). Thus, equality (3.1), as suggested in the original arrangement, describes the behavior of structures under conditions of external dry friction, but not constantly and with weak inhomogeneity: $\tan\varphi \sim 1/r$. In a real shell, external friction at the contact of layers is conveniently prescribed as constant. It is clear that construction of solution (3.2) and (3.4) from a solution with $\varphi = \text{const}$ differs insignificantly, which in turn makes it possible to avoid solving the quite cumbersome problem with constant friction at the contact.

Normally condition (3.1) is used in order to describe the behavior of material with internal friction (loose materials, rocks, etc.). Consideration of adhesion in (3.1) does not markedly complicate the problem. For these materials coefficient f < 1. In mathematical models, the condition f < 1 provides a hyperbolic statement of the problem [3], and therefore the case of f > 1 is not considered, due to its physical unreality and the elliptical nature of the mathematical arrangement. As follows from (3.4), much more extensive possibilities develop in a shell. In particular, since here free parameter $0 \leqslant \varphi < \pi/2$, it is possible to realize structurally stressed state (3.2) with coefficient f within the limits

$$0 \leqslant f < 1/|\cos 2\varkappa|, \text{ if } \pi/4 \leqslant \varkappa \leqslant \pi/2;\ 0 \leqslant f < \infty,\ -\infty < \\ < f \leqslant -1/|\cos 2\varkappa|, \text{ if } 0 \leqslant \varkappa \leqslant \pi/4. \tag{3.5}$$

In future we limit ourselves only to equalities (3.5).

We turn to finding the kinematics corresponding to stresses (3.2). By substituting (3.2) in set (1.2) we obtain

$$w_1^0 = -pR_0^{2f/(1+f)} \frac{1}{2\mu(1+f)} ((1-2\nu+f)J_1 - 2fJ_2) + g_2(\lambda_2),$$

$$w_2^0 = -pR_0^{2f/(1+f)} \frac{1}{2\mu(1+f)} \Big(\frac{2f}{\xi} \int J_2\, da_2 - \frac{1-2\nu+f}{\xi} \int J_1\, da_2 + \tag{3.6}$$

$$+ \frac{1-2\nu-f}{2\xi}(1+f)(a_2^2+\xi^2)^{1/(1+f)} - \xi(1+f)(a_2^2+\xi^2)^{-f/(1+f)} \Big) - \int g_2(\lambda_2)\, d\lambda_2 + g_1(\lambda_1),$$

where $J_1 = \int (a_2^2+\xi^2)^{-f/(1+f)} da_2$; $J_2 = \xi^2 \int (a_2^2+\xi^2)^{-(1+2f)/(1+f)} da_2$; arbitrary integration functions $g_i(\lambda_i)$ (i = 1, 2) are determined from boundary conditions of type (2.6). Thus, (3.2), (3.4), (3.6) provide the solution of direct problem (1.1), (1.2), (3.1), and from (1.3), function Γ is found relating to the solution.

We consider the problem of optimizing and fixing external pressure q ≠ 0. Then according to (3.3),

$$p = q(R/R_0)^{2f/(1+f)}. \tag{3.7}$$

Apart from (3.7), from the requirements for structural integrity there emerges a limitation on internal pressure in the form

$$p < \tau_s(1+f)/f. \tag{3.8}$$

Thus, the best performance criterion in this situation means the following: it is necessary to find a value f = const which would satisfy inequality (3.8) and lead to the maximum p in equality (3.7).

If the structure is designed to operate with $q < \tau_s(R_0/R)^2$, then the best performance criterion leads to the situation when $f \to \infty$ and the maximum possible internal pressure in this case $p \to \tau_s$. Thus, with quite small values of q, the supporting capacity of a rolled shell differs little from the supporting capacity of a one-piece thick-walled tube. For the class of loading with $q > \tau_s(R_0/R)^2$ the best performance condition leads a value of f satisfying the relationship $\frac{q}{\tau_s}\frac{f}{1+f}\left(\frac{R}{R_0}\right)^{2f/(1+f)} = 1$, and the maximum internal pressure $p = \tau_s(1 + f)/f$.

We illustrate the results obtained with an example comparing the supporting capacity of two structures: a thick-walled tube $(R_0 \leqslant r \leqslant R)$ and a rolled shell $(R_0 \leqslant r \leqslant c)$ with an external elastic ring $(c \leqslant r \leqslant R)$. Let pressure at the outer boundary be absent $(\sigma_r|_{r=R} = 0)$. Then, as is well known [2], in the first case with retention of material integrity, the maximum possible pressure is

$$p_1 = \tau_s(1 - R_0^2/R^2), \tag{3.9}$$

and $p \to \tau_s$ if $R \to \infty$, i.e., with an unlimited increase in thickness, its supporting capacity remains limited. In analyzing the supporting capacity of the second structure parameter c appears which can be used in addition to coefficient f, as an optimizing parameter.

In the end, optimum values of c_* and f_* will be solutions of equations $\frac{c_*^2}{R^2} = \frac{\ln(R/c_*)}{\ln(R/R_0)}$ and $f_* = c_*^2/(R^2 - 2c_*^2)$. With these f_* and c_*, the maximum possible pressure reaches a value

$$p_2 = \tau_s(R^2/c_*^2 - 1). \tag{3.10}$$

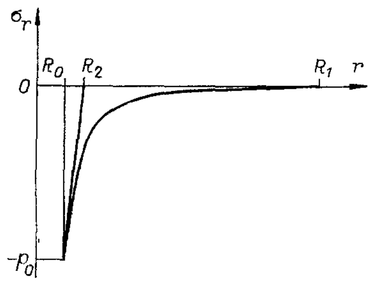

Fig. 2

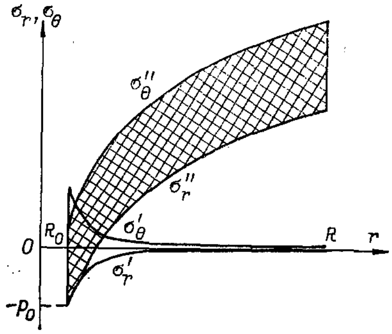

Fig. 3

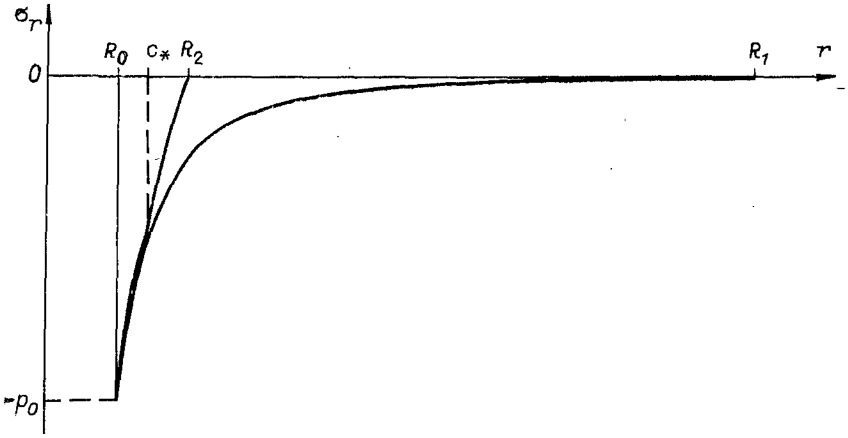

Fig. 4

It is easy to show that $p_2 \to \infty$ as $R \to \infty$, i.e., an unlimited increase in the thickness of a rolled tube with an external elastic ring increases its supporting capacity also in an unlimited way.

It follows from (3.9) and (3.10) that if a certain internal pressure $p_0(p_0 = 0.99\ \tau_s)$ is fixed, then with constant R_0, the minimum external radius of a one-piece tube $R_1 = R_0/\sqrt{1 - p_0/\tau_s}$ ($R_1 = 10\ R_0$), whereas the required external radius for a rolled shell with an elastic ring is markedly less: $R_2 = R_0 \exp((1/2)(1 + p_0/\tau_s) \ln(1 + p_0/\tau_s))$ ($R_2 \approx 1.98R_0$).

Thus, the results obtained indicate that the stressed state of a structure may be controlled by means of selecting the optimum reaction of layers. The material loading achieved is more uniform than in a one-piece tube, which makes it possible to increase the supporting capacity of the strucutre by a factor of two to four.

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